Carbon Dioxide, $T_0 \approx 295^0K$

	Orifice Diam, cm	x_s/d	Velocity	Skimmer 20⁰K	Skimmer 295⁰K	Mass Spectrometer or Multiplier, Side-On	Mass Spectrometer or Multiplier, End-On
1	0.0386	1000	Monomer	x		x	
2	'	800	Monomer		x	x	
3		1000	Monomer	x			x
4		800	Monomer		x		x
5		1000	Total	x			x
6		800	Total		x		x
7	0.049	8	Total		x		x
8	0.030	10	Total		x		x
9	Limiting Thermal Velocity						

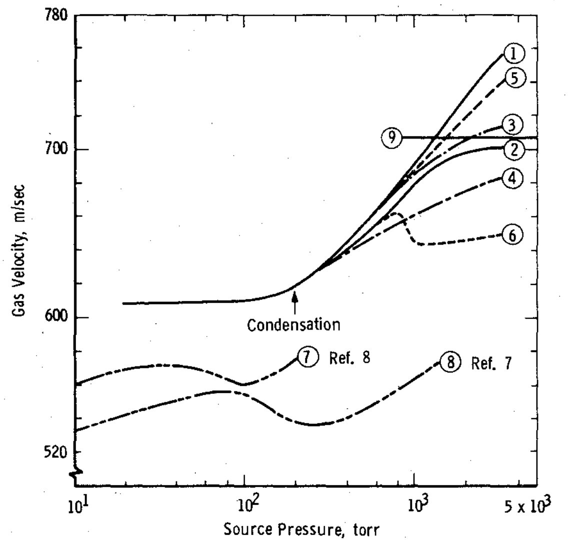

Fig. 2 Effect of skimmer interaction on measured beam velocity.

creases with increasing source pressure. It can be seen from Fig. 1 that the total beam intensity for configurations B, C, and D approaches to within 40% of configuration A at the highest source pressure.

A series of velocity measurements has been made in a carbon dioxide molecular beam for a number of skimmer and velocity detector configurations (Fig. 2). Using the criterion discussed earlier, we assumed that there are clusters in the flow for source pressures greater than 150 torr. In noncondensed flow the present results indicate that, for large source skimmer separations, i.e., $x_s/d > 800$ the measured velocity is independent of the skimmer configuration. Velocity measurements[7,8] at $8 < x_s/d < 10$ are lower than the present data and presumably reflect a skimmer and/or endwall interference effect upon beam velocity.

In condensed flows the measured velocity is dependent upon the skimmer configuration used to form the molecular beam (e.g., Fig. 2). There is experimental evidence[9] to suggest that at these source-flow conditions, condensation occurs within a few nozzle diameters of the source. These large clusters are formed before the noncondensed gas has attained the limiting thermal velocity. Abuaf et al.[10] have shown that, in the expansion of gas mixtures composed of heavy and light molecules for a range of source conditions, the velocity of the heavy molecule can be less than that of the light molecule. On the basis of these measurements, it is assumed the large clusters formed in the carbon dioxide expansion are traveling slower than the noncondensed gas. Fragmentation of molecular clusters can occur in the ionizing region of a mass spectrometer. Therefore, the monomer intensity is comprised of freestream monomers and fragmentation monomers. From an earlier consideration of skimmer effects it can be concluded that freestream monomers are dependent upon the skimmer configuration, whereas those monomers resulting from the fragmentation of large clusters are not. This results in an increase in the ratio of slow to fast monomers in a molecular beam formed with a 295 K skimmer over that formed by a 20 K skimmer. This in turn results in a lower average velocity for the 295 K skimmer (e.g., Fig. 2). Note that with the improved technique of cryogenic skimming it is possible to observe the expected increase in beam velocity over the limiting thermal velocity, due to the release of the heat of condensation.

References

[1] Wegener, P.P., ed., *Molecular Beams and Low-Density Gas Dynamics*, Marcel Dekker, New York, 1974.

[2] Golomb, D., Good, R. E., and Brown, R. F., "Dimers and Clusters in Freejets of Argon and Nitric Oxide." *Journal of Chemical Physics*, Vol. 52, Feb. 1970, pp. 1545-1551.

[3] Golomb, D., Good, R.E., Bailey, A.B., Busby, M.R., and Dawbarn, R., "Dimers, Clusters, and Condensation in Freejets II," *Journal of Chemical Physics*, Vol. 57, Nov. 1972, pp. 3844-3852.

[4] Audit, Phillippe, "Liaisons Intermoleculaires dans les Jets Supersoniques Etude par Diffraction d'Electrons" *Le Journal de Physique*, Vol. 30, Feb.-March 1969, pp. 192-202.

[5] Sherman, P.M., "Condensation Augmented Velocity of a Supersonic Stream," *AIAA Journal*, Vol. 9, Aug. 1971, pp. 1628-1630.

[6] Leckenby, R.E., Robbins, E.J., and Trevalion, P.A., "Condensation Embryos in an Expanding Gas Beam," *Proceedings of the Royal Society of London*, Vol. 280, Ser. A, March 1964, pp. 409-429.

[7] Hagena, O. and Henkes, W., "Untersuchung der Thermischen Relaxation bei Dusentromungen Durch Analyse der Gaskinetschen Geschwindigekeitsverteilung." Zeitschrift Naturforsch, Vol. 15a, 1960, pp. 851-858.

[8] Bier, K. and Hagena, O., "Influence of Shock Waves on the Generation of High-Intensity Molecular Beams by Nozzles," *Proceedings of the Third International Rarefied Gas Dynamics Symposium*, Academic Press, New York, 1963, pp. 478-496.

[9] Lewis, J.W.L. and Williams, W.D., "Argon Condensation in Freejet Expansions," AEDC-TR-74-32, July 1974, Arnold Engineering Development Center, Arnold Air Force Station, Tenn.

[10] Abuaf, N., Anderson, J.B., Andres, R.P., Fenn, J.B., and Miller, D.R., "Studies of Low-Density Supersonic Jets," *Proceedings of the Fifth International Rarefield Gas Dynamics Symposium*, Academic Press, New York, 1967, pp. 1317-1336.

Buckling of Cylinders of Variable Thickness under Lateral Pressure

S. K. Radhamohan*
Vikram Sarabhai Space Center, Trivandrum, India
and
A. V. Setlur†
Pioneer Service Engineering Company, Chicago, Ill.

Introduction

SITUATIONS arise where it is necessary to join two shells of unequal thickness or radius of curvature. Significant bending stresses and local variations of membrane stresses are known to exist at the junction of these shells. It is customary to strengthen the structure in this region by gradually increasing the thickness. Not much attention has been given to the analysis of shells of variable thickness, particularly in the stability domain. The purpose of this Note is, therefore, to bridge the gap and compare the efficiency of these shells with shells of constant thickness from buckling considerations.

Federhofer[1] and Wagner[2] treated axisymmetric buckling of cylinders of linearly varying thickness. Asymmetric buckling of a stepped cylinder under external pressure was analyzed by

Received November 14, 1974; revision received July 22, 1975.

*Engineer, Structural Engineering Division.

†Engineer.

Ross,[3] using a simple sine wave form for the buckled shape of a simply supported cylinder. This approximate shape led to an overestimate of the buckling pressure.

In this paper, a detailed buckling analysis of cylinders of linearly varying thickness is undertaken for the first time. Emphasis is given to study the effects of 1) thickness variation, 2) length to radius ratio, 3) prebuckling deformations and stresses and 4) different boundary conditions on the buckling behavior of cylinders subject to external uniform pressure.

Theory

The differential equations governing the prebuckled and buckled states of a thin shell of revolution are derived from Sanders' nonlinear shell theory[4] by using adjacent equilibrium approach. The constitutive relations, corresponding to a homogeneous and isotropic material, are chosen. It is assumed that the external loading acts at the middle of the shell. "Parametric Differentiation" technique coupled with segmentation scheme is employed to solve the nonlinear differential equations of the prebuckled state, which is axisymmetric. A detailed description of the governing equations and the method of solution can be found in Refs. 5 and 6. The thickness variation is taken as: $\bar{t}=(A_I+B_I\bar{s})$ for $0\leq\bar{s}\leq 1$ (where $\bar{s}$ is the distance from one edge of the shell and is nondimensionalized with respect to half the length of the shell; A_I, B_I are the parameters governing the thickness variation; $\bar{t}$ is the nondimensionalized thickness). Shells having two types of thickness variation are studied (Fig. 1). The average thickness (or equivalently the weight of the structure) is kept constant, and the efficiency of these shells, from buckling point of view, is compared with shells of constant thickness (corresponding to the average value).

Numerical Results

The uniform external pressure at the middle surface of the cylinder is

$$q=2E/[3(1-\nu^2)]^{1/2}\,(t_{av}/R)^2\lambda \tag{1}$$

where q=external pressure, E,ν=Youngs modulus and Poisson's ratio, R,t_{av}=radius and average thickness, respectively, and λ=load parameter. The geometric parameter $\bar{Z}_0$ is chosen as

$$\bar{Z}_0=L^2/Rt_{av}[1-\nu^2]^{1/2} \tag{2}$$

where L is length of the shell.

Classical boundary conditions, namely $w_n^*=M_{sn}^*=v_n^*=N_{sn}^*=0$, are adopted to study the effect of geometric and thickness parameters.

Effect of Thickness Variation

Critical pressures of a cylinder ($\bar{Z}_0=1000$) having different thickness distribution are listed in Table 1. Shells which are thin at the ends are found to resist higher pressures than those shells with thick ends. From the results, it appears that the circumferential wave number is not altered by the type of thickness variation.

Effect of L/R Ratio

Buckling parameters ($\bar{K}_p=qR\,L^2/\pi^2D$) for shells of constant thickness are given in Ref. 7. In this section, the parameter $\bar{K}_p$ for shells having minimum thickness at the ends, is evaluated ($A_I=1/2$, $B_I=1.0$). The logarithmic plot between $\bar{K}_p$ and $\bar{Z}_0$ is shown in Fig. 2. The magnitude of $\bar{K}_p$ increases for increasing values of $\bar{Z}_0$. The deviation of $\bar{K}_p$ between shells of constant and variable thicknesses, is larger as $\bar{Z}_0$ increases. The relation between $\bar{K}_p$ and $\bar{Z}_0$ for higher $\bar{Z}_0$ values can be approximated as

$$\bar{K}_p=1.39\bar{Z}_0^{0.53} \tag{3}$$

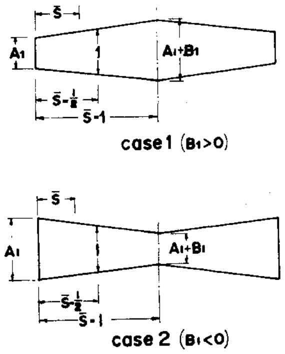

Fig. 1 Thickness variation.

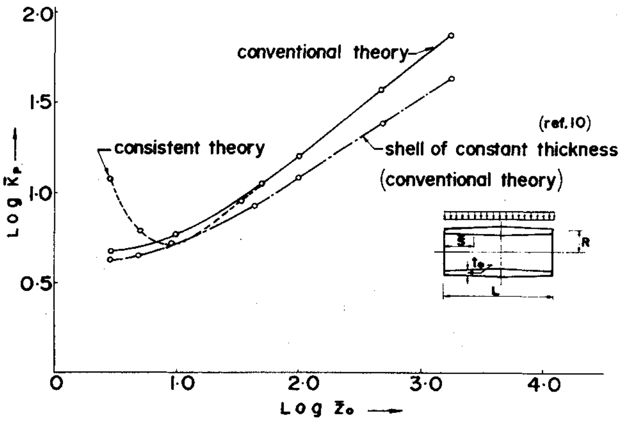

Fig. 2 Buckling pressures of laterally loaded cylinders.

A similar relation for shells of constant thickness is given[7] as

$$\bar{K}_p=1.04\bar{Z}_0^{1/2} \tag{4}$$

Effect of Prebuckling Deformation and Stresses

This effect, for a shell having thickness parameters ($A_I=1/2$, $B_I=1.0$), is shown in Fig. 2. For values of $\bar{Z}_0<7.1$, consistent theory predicts higher values than membrane prebuckling analysis. In between $7.1\leq\bar{Z}_0\leq 140$, consistent theory estimates lower values. Beyond this value the two approaches lead to the same buckling pressure. For very low

Table 1 Buckling pressures of a cylinder for different thickness variations

($R/t_0=100.0$, $L/t_0=325$, $\nu=0.3$)

S. No.	Thickness coefficients	λ_{cr}	n	Mode of buckling
1	$A_I=0.5$, $B_I=1.0$	0.0428	4	Symmetric
		0.03985	5	
		0.0464	6	
2	$A_I=0.7$, $B_I=0.6$	0.03799	4	Symmetric
		0.03327	5	
		0.04048	6	
3	$A_I=1.0$, $B_I=0.0$	0.0318	4	Symmetric
		0.025	5	
4	$A_I=1.2$, $B_I=-0.4$	0.02408	4	Symmetric
		0.0235	5	
		0.02797	6	

Table 2 Critical pressures for cylinder for different boundary conditions

Type of boundary condition	Critical pressure n	λ_{cr}
Eq. (5a)	5	0.0371
	6	0.04192
Eq. (5b)	5	0.03985
	6	0.0464
Eq. (5c)	5	0.04555
	6	0.04937
Eq. (5d)	5	0.05203
	6	0.05465

values of $\bar{Z}_0$, the shell buckles into more number of circumferential waves, if membrane prebuckling is employed.

Effect of Buckling Boundary Conditions

The possible set of simply-supported boundary conditions are

$$W_n^* = M_{sn}^* = N_{sn}^* = N_{s\theta n}^* = 0 \tag{5a}$$

$$W_n^* = M_{sn}^* = N_{sn}^* = V_n^* = 0 \tag{5b}$$

$$W_n^* = M_{sn}^* = U_n^* = N_{s\theta n}^* = 0 \tag{5c}$$

$$W_n^* = M_{sn}^* = U_n^* = V_n^* = 0 \tag{5d}$$

For the previously listed boundary conditions, critical loads of a shell whose thickness variation corresponds to S. No. 1 of Table 1 are given in Table 2. A shell whose inplane displacements are restrained completely, buckles at higher pressures than a shell having zero inplane edge forces. Critical pressures for the other two boundary conditions fall within the aforementioned limits.

Conclusions

The numerical results presented here show that shells of variable thickness having particular thickness variation resists higher lateral pressures than shells of constant (average) thickness for an assumed set of boundary conditions. Critical pressures of short shells, based on consistent prebuckling analysis, are larger than membrane analysis.

References

[1] Federhofer, K., "Stabilitat Der Krieszylinderschale Mit Veranderlicher Wandstarke," *Osterreichisches*, Ingenieur—Archiv, Vol. 6, 1952, pp. 277-288.

[2] Wagner, H., "Die Stabilitat Der Axial Gendruckten Kreiszylinderschale Mit Veranderlicher Wandstarke," *Osterreichischers*, Ingenieur-Archiv, Vol. 13, 1959, pp. 235-257.

[3] Ross, C. T. F., "Elastic Instability of a Circular Cylinder of Varying Shell Thickness," *Quarterly Transactions of Royal Society for Naval Architects*, Vol. 110, April 1968, pp. 247-290.

[4] Sanders, J.L. "Nonlinear Theories for Thin Shells," *Quarterly of Applied Mathematics*, Vol. 21, April 1963, pp. 21-36.

[5] Radhamohan, S. K., "Nonlinear Stability of Thin Shells of Revolution by Parametric Differentiation Technique," Ph. D. thesis, 1971, Dept. of Civil Engineering, Indian Institute of Technology, Kanpur, India.

[6] Radhamohan, S. K. and Prasad, B., "Axymmetric Buckling of Toroidal Shells under Axial Tension," *AIAA Journal*, Vol. 12, April 1974, pp. 511-515.

[7] Batdorf, S. B., "A Simplified Method of Elastic Stability Analysis for Thin Cylindrical Shells—I—Donnell's Equations," June 1947, NACA TN-1341.

Debris Shielding in Regions of High Edge Velocity

David H. Smith*
Prototype Development Associates, Inc., Santa Ana, Calif.

Nomenclature

A	= area, ft^2
B	= mass loss constant, sec^2/ft^2
C	= trajectory constant, 1/ft
C_D	= drag coefficient
D	= particle diameter, ft
e	= base of natural logarithms
G	= ratio of eroded-to-impact mass,
m	= mass, lb
$\dot{m}$	= incident mass flux, lb/ft^2-sec
$P(n)$	= probability of n collisions
r	= radius, ft
s	= running length, ft
t	= time, sec
V	= velocity, ft/sec
ρ	= density, lb/ft^3
θ	= angle from velocity vector ($\theta_{stag} = 90$), degree
Subscripts	
b	= body
d	= debris
e	= boundary-layer edge
I	= impact
o	= original
p	= particle
rel	= relative

Introduction

It has been established,[1] that for sufficiently high particle fluxes, 1) the erosion of the stagnation region of a blunt body traversing a particle cloud at hypersonic velocity is a nonlinear function of the particle impacting mass flux, and 2) this nonlinearity can be explained by debris shielding. Some incoming particles will collide with debris fragments created by upstream collisions. Due to the large difference in velocity between the particle and debris fragment, these collisions reduce the surface impact damage and, in effect, "shield" the surface. The majority of the work on debris shielding to date has been for stagnation regions. This note attempts to extend debris shielding analysis to regions with super-or hypersonic edge velocity.

The probability of debris-particle collisions and the resultant damage are studied for regions of high edge velocity. It is shown that in such regions debris shielding for typical reentry conditions is dominated by debris created only a few inches upstream of any point under analysis.

Analytical Methods

The computer program used for this analysis calculates ablation, erosion, and shape change for hypersonic vehicles. Flow parameters are calculated by a streamwise numerical integration so that the effects of the bow-shock-generated entropy layer are accounted for by a streamtube mass balance. In the erosion calculations[2] particle trajectories are calculated from the shock to the body by coupling the effects of

Received February 18, 1975; revision received June 6, 1975. This work was sponsored by the Defense Nuclear Agency under contract, DNA 001-75-C-0054.

Index categories: Multiphase Flows; Hypervelocity Impact.

*Engineering Associate.